## CUBE AND CUBE ROOTS

**Cube number or Perfect cubes:** It is a number which is the product of three same numbers.

*Example:* Cube number of 2 will be 2 x 2 x 2 = 8. Thus, 8 is a cube number.

Cubes of some natural numbers:

| Number | Cube | Number | Cube |
|---|---|---|---|
| 1 | 1 x 1 x 1 = 1 | 11 | 11 x 11 x 11 = 1331 |
| 2 | 2 x 2 x 2 = 8 | 12 | 12 x 12 x 12 = 1728 |
| 3 | 3 x 3 x 3 = 27 | 13 | 13 x 13 x 13 = 2197 |
| 4 | 4 x 4 x 4 = 64 | 14 | 14 x 14 x 14 = 2744 |
| 5 | 5 x 5 x 5 = 125 | 15 | 15 x 15 x 15 = 3375 |
| 6 | 6 x 6 x 6 = 216 | 16 | 16 x 16 x 16 = 4096 |
| 7 | 7 x 7 x 7 = 343 | 17 | 17 x 17 x 17 = 4913 |
| 8 | 8 x 8 x 8 = 512 | 18 | 18 x 18 x 18 = 5832 |
| 9 | 9 x 9 x 9 = 729 | 19 | 19 x 19 x 19 = 6859 |
| 10 | 10 x 10 x 10 = 1000 | 20 | 20 x 20 x 20 = 8000 |

**Properties of Cube Numbers:**

**1. The cube of an even number will always be an even number.**

*Example*: $8^3 = 512$, $12^3 = 1728$, etc.

**2. The cube of odd number will always be an odd number.**

*Example*: $7^3 = 343$, $19^3 = 6589$, etc.

**3. If the cube number have x at its one's digit or unit's place then it always end with the digit as shown in the table below:**

| Unit's digit of number | Last digit of its cube number | Example |
|---|---|---|
| 1 | 1 | $11^3 = 1331$, $21^3 = 9261$, etc. |

| | | |
|---|---|---|
| 2 | 8 | $2^3 = 8, 12^3 = 1728, 32^3 = 32768$, etc. |
| 3 | 7 | $13^3 = 2197, 53^3 = 148877$, etc. |
| 4 | 4 | $24^3 = 13824, 74^3 = 405224$, etc. |
| 5 | 5 | $15^3 = 3375, 25^3 = 15625$, etc. |
| 6 | 6 | $6^3 = 216, 26^3 = 17576$, etc. |
| 7 | 3 | $17^3 = 4913, 37^3 = 50653$, etc. |
| 8 | 2 | $8^3 = 512, 18^3 = 5832$, etc. |
| 9 | 9 | $19^3 = 6859, 39^3 = 59319$, etc. |
| 10 | 20 | $10^3 = 1000, 20^3 = 8000$, etc. |

***Example 1***: Find the one's digit for 27.

*Solution*: As the last digit of given number is 7, So the one's digit for 27's cube number will be 3.

***Example 2***: Find the one's digit for 149.

*Solution*: As the last digit of given number is 9, So the one's digit for 149's cube number will be 9.

**Interesting patterns of Cube Number:**

**1. Addition of consecutive odd numbers will give Cube Number-**

$1^3 = 1 = 1$

$2^3 = 8 = 3 + 5$

$3^3 = 27 = 7 + 9 + 11$

$4^3 = 64 = 13 + 15 + 17 + 19$

$5^3 = 125 = 21 + 23 + 25 + 27 + 29$

**2. Cubes and their prime factors-**

The prime factors of any cube number will be in pair of 3.

*Example*:

(i) $4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$

(ii) $12^3 = 1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 2^3 \times 3^3$

*Note:*

(i) If any prime factor is not in pair of 3, then it will not be a perfect cube.

(ii) Numbers like 1729, 4104, 13832, are known as Hardy – Ramanujan

(iii) Numbers. They can be expressed as sum of two cubes in two different ways.

***Example 1***: Is 128 a perfect cube number?

*Solution*: After finding prime factors of 128, we can write $128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$

We can see that each prime factor is not in pair of 3. Hence, 128 is not a perfect cube number.

***Example 2***: Find the smallest number by which 675 must be multiplied to obtain a perfect cube.

*Solution*: On finding prime factors of 675, we have $675 = \underline{3 \times 3 \times 3} \times 5 \times 5$.

We can see that, triplets of 5 is missing. Hence, on multiplying given number by 5 we can have a perfect cube number. Thus, $675 \times 5 = 3375$ which is a perfect cube number.

***Example 3***: Find the smallest number by which 192 must be divided to obtain a perfect cube.

*Solution*: On finding prime factors of 192, we have $192 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$.

We can see that, triplets of 3 is missing while other numbers have triplets. Hence, on dividing given number by 3 we can have a perfect cube number.

Thus, $192 / 3 = 64$ which is a perfect cube number.